Interplay between Wishart and GIG matrices

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Abstract

While the Wishart distribution is classical for multivariate analysis, the matrix variate generalized inverse Gaussian (MGIG) distribution is relatively new. It was introduced by Barndorff-Nielsen, Blæsild, Jensen, and Jørgensen in 1982 in the context of exponential transformation models. Connections between Wishart and MGIG distributions were observed, rather separately, by Butler in 1998 and Bernadac in 1995. A much better understanding of relations between these distributions can be achieved through, so called, Matsumoto-Yor (MY) property. This property was originally discovered in a univariate setting about 1999 (the paper by Matsumoto and Yor appeared in 2001) during investigations of functionals of exponential Brownian motion. Later, in 2000, its matrix version was derived. Here we will be mostly interested in consequences of MY property for matrices of different dimensions. It appears that this property allows to unify Butler's and Bernadac's results. In particular, it will be explained, how the MY property can be imbedded in the conditional structure of Wishart matrices. Additionally, a mixture Wishart-MGIG model will be analyzed briefly. The talk is based on investigations done during last several years jointly with G. Letac (Toulouse), H. Massam (Toronto), V. Seshadri (Montreal).

References

Barndorff-Nielsen, O., P. Blæsild, J.L. Jensen, and B. Jørgensen. (1982) Exponential transformation models. *Proc. Roy. Soc. Lond. A 379*, 41–65.

Bernadac, E. (1995). Random continued fractions and inverse gaussian distribution on a symmetric cone. *J. Theor. Probab. 8*, 221–259.

Butler, R.W. (1998). Generalized inverse Gaussian distributions and their Wishart connections, *Scand. J. Statist. 25*, 69–75.

Letac, G. and J. Wesołowski (2000). An independence property for the product of GIG and gamma laws. *Ann. Probab. 28*, 1371–1383.

Massam, H. and J. Wesołowski (2006). The Matsumoto-Yor property and the structure of the Wishart distribution. *J. Multivar. Anal. 97*, 103–123.

Matsumoto, H. and M. Yor (2001). An analogue of Pitman's $2M - X$ theorem for exponential Wiener functionals. Part II: The role of the generalized inverse Gaussian laws, *Nagoya Math. J. 162*, 65–86.

Seshadri, V. and J. Wesołowski (2007). More on connections between Wishart and matrix GIG distributions, *Metrika* - to appear.