How to analyze variety trials without a computer?

L. Rob Verdooren

Numico Research B.V. Wageningen, The Netherlands

Abstract

In the years after the Second World War many variety trials were performed. If the number of varieties was not too large, Randomized Complete Block Designs were used. When the number of varieties was large, only incomplete homogeneous blocks could be used. The nicest design to replace the Randomized Complete Block Design was the Balanced Incomplete Block Design, but such a design requires too much replications. Usually three or four replications of the varieties were used in the variety testing.

To fit the needs of the plant-breeders for variety trials with three or four replications with incomplete blocks, Lattice Designs, Rectangular Lattices, Partially Balanced Incomplete Block Designs with two associate classes, and later also Cyclic Designs were used. For the analysis of these designs formulas were available to calculate the Analysis of Variance (ANOVA) table together with the F-test for the test of equal varietal effects, and to get the Best Linear Unbiased Estimator (BLUE) for the varietal contrasts, together with their Standard Errors.

But the published incomplete block designs were as a Procrustes bed for the plant-breeders. When the number of varieties to be tested was larger than the number of varieties for the published design, they must left out a number of varieties. If the number of varieties was less than the number of varieties of the published design, they must use some varieties more often as a pseudo different variety, to increase the number of varieties. But in this last case there were no formulas available to do the correct analysis. Also in the case that there were missing plots in an incomplete block design, there were no exact formulas available for the correct analysis. Also when a plant-breeder construct himself an incomplete block design, the correct analysis was not found.

Of course the theoretical solution was known. The linear model could be described as

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e},$$

where $\mathbf{X}$ is the design matrix and b is the parameter-vector. Solve the normal equations,

$$(\mathbf{X}'\mathbf{X})\,\mathbf{b} = \mathbf{X}'\mathbf{y},$$

and the variance-covariance matrix of the estimator

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{X}'\mathbf{y}$$

is $\sigma^2\,(\mathbf{X}'\mathbf{X})^{-1}$. But the calculation of the inverse of the matrix $(\mathbf{X}'\mathbf{X})$ is cumbersome to do it practically by hand. In that time there were no computers available which could invert matrices as large as e.g. 40 x 40 .

In 1948 Stevens published a general iterative procedure to get a solution of b. The correctness of his iterative procedure that it always converge, was given in 1952 by Kuiper for a two-way layout, and in 1958 Corsten proved the convergence of the iterative procedure for a three-way layout. The drawback of the iterative procedure was, that only two goals of the analysis were reached, namely the ANOVA table and the BLUE for varietal contrasts. But the Standard Error of the BLUE for a varietal contrast was not given.

In 1971 Caliński published the solution for the calculation of the Standard Error of the BLUE for a varietal contrast, by using the same iterative procedure. The iterative procedure will be described to get a solution for **b**, the calculation of the ANOVA table with the F-test for varieties and due to Caliński the calculation of the Standard Error of a BLUE for a varietal contrast.