Relaxed alternating projection methods

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Abstract

Let A and B be nonempty, convex and closed subsets of a Hilbert space $\mathcal{H}$. In the practical considerations we need to find an element of the intersection $A \cap B$ or, more general, to solve the following problem:

$$\text{find } a^* \in A \text{ and } b^* \in B \text{ such that } \|a^* - b^*\| = \inf_{a \in A, b \in B} \|a - b\|.$$

One of the important methods generating sequences which converge weakly to a solution of above problems is the von Neumann alternating projection method $x_{k+1} = P_A P_B x_k$. The method has found application in different areas of mathematics. These include probability and statistics, image reconstruction and intensity modulated radiation therapy, where the convex subsets are described by a large and sparse system of linear equations or inequalities.

We deal with generalization of the von Neumann alternating projection method of the form $x_{k+1} = P_A(x_k + \lambda_k(x_k)(P_A P_B x_k - x_k))$, where $\operatorname{Fix} P_A P_B \neq \varnothing$. We give sufficient conditions for the weak convergence of the sequence (x_k) to $\operatorname{Fix} P_A P_B$ in the general case and in the case A is a closed affine subspace. We present also the results of preliminary numerical experiments.

Keywords

Alternating projection method, Fejér monotonicity, Weak convergence.

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