Relaxed alternating projection methods

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Abstract

Let $A$ and $B$ be nonempty, convex and closed subsets of a Hilbert space $H$.
In the practical considerations we need to find an element of the intersection
$A \cap B$ or, more general, to solve the following problem:

$$
\text{find } a^* \in A \text{ and } b^* \in B \text{ such that } \|a^* - b^*\| = \inf_{a \in A, b \in B} \|a - b\|.
$$

One of the important methods generating sequences which converge weakly
to a solution of above problems is the von Neumann alternating projection
method $x_{k+1} = P_A P_B x_k$. The method has found application in different
areas of mathematics. These include probability and statistics, image re-
construction and intensity modulated radiation therapy, where the convex
subsets are described by a large and sparse system of linear equations or
inequalities.

We deal with generalization of the von Neumann alternating projection
method of the form $x_{k+1} = P_A(x_k + \lambda_k(x_k)(P_A P_B x_k - x_k))$, where
Fix $P_A P_B \neq \emptyset$. We give sufficient conditions for the weak convergence of
the sequence $(x_k)$ to Fix $P_A P_B$ in the general case and in the case $A$ is a
closed affine subspace. We present also the results of preliminary numerical
experiments.

Keywords
Alternating projection method, Fejér monotonicity, Weak convergence.

References


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