Tests of additivity in mixed and fixed effects two-way ANOVA models with single subclass numbers

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Abstract

In **variety testing** a relative large number ν (30 or more) of varieties has to be tested in b blocks, each with relative small number k of varieties. If $\nu = k$ a complete block design is used but usually with just one observation for each variety x block combination. Varieties can be considered as the levels of a fixed factor whereas the blocks are usually considered as randomly selected levels from a population of possible blocks.

In **psychological tests** a relative large number ν (50 or more) of items has to be tested at b (100 or more) individuals (testees). As the result we have just one observation for each item x testee combination, namely "correct = 1" or "wrong = 0". Items can be considered as the levels of a fixed factor whereas the testees are usually considered as randomly selected from a population of possible testees.

The analyses in both cases are done under the assumption of missing interactions between the fixed and random factor. Tests of the no interaction are known only for (two) fixed factors. First results of tests developed by the authors in the mixed model situation are presented in the sequel.

First a test of an interaction in the block design is considered. It is difficult because of no replications within subclasses. In this situation it is not possible to separate interaction from the error term. Tukey (1949) introduced some restrictions to the structure of interaction, in particular that the interaction effect τ_{ij} is the product of corresponding factor levels times a shift factor: $\tau_{ij} = \lambda a_i \beta_j$ (α_i effect of the factor, $i = 1, \ldots, a$; β_j block effect, $j = 1, \ldots, b$). This means, that the interaction is the product of factor and block effects. Similar restrictions can be found in Mandel (1961) and Johnson and Graybill (1972). In the proposed method no constraint to the structure of interaction are made, but there are some restriction to the design of the experiment. A well known design which allows estimation of

these interactions is latin square. But in addition to the ordinary assumption that $\sum \gamma_i = 0$ (γ_i column effect, $i = 1, , a = b$), we also assume that all interaction effects within a column sum up to zero. This is very intuitive, as this is the common assumption for interactions within blocks. As there is no real distinction between blocks and columns this assumption seems to be reasonable. As simulations showed, power of the proposed method is high and it is relative robust against violations of prerequisites. The method as such however is not restricted to latin squares but can be generalized to a very broad range of experimental designs.

Next, five known tests of interaction in the model without replication are verified for the using in the mixed model. Tukey's, Mandel's, Johnson Graybill's, locally best invariant (LBI) and Tusell's test are concerned on the level 5 %. Simulation was performed to examine whether the type-I-risk remains on 5 % level even for mixed ANOVA models. The number of levels of the fixed factor was chosen between 3 and 10, of the random factor between 4 and 50, the variance of the random factor equals 2, 5 or 10, the variance of the random error equals 1. It was concluded that for 5 % type-I-risk all these tests hold the level of type-I-risk and therefore the tests developed for fixed models can be used for the mixed models as well.

We also discuss the consequence of only one observation per cell in the testing of interaction. A modification of Tukey's additivity test is derived. Then in a simulation study we show that when the interaction is a product of the main effects, the power of the modified test appears to be similar to the power of Tukey's or Mandel's test and outperforms the Johnson–Graybill, locally best invariant (LBI) and Tussel's test. When the interaction scheme is more general the power of the modified test is not as good as for the Johnson–Graybill, LBI and Tussel's test but is still much better than Tukey's and Mandels tests.

Finally, the two-way ANOVA model without replication with binary outcome is an important problem, especially in the psychological research. Some properties of the test of the interaction in this case are discussed.

Keywords

Two-way ANOVA without replication, Additivity tests, Mixed model, Block design.

References

Alin, A. and S. Kurt (2006). Testing non-additivity (interaction) in two-way ANOVA tables with no replication. *Stat. Meth. in Med. Res. 15*, 63–85.

Boik, R.J. (1993a). Testing additivity in two-way classifications with no replications: the locally best invariant test. *J. Appl. Stat. 20*, 41–55.

Boik, R.J. (1993b). A comparison of three invariant tests of additivity in two-way classifications with no replications. *Comput. Statist. Data Anal. 15*, 411–424.

Corsten, L.C.A. and C.A. van Eijnsbergen (1972). Multiplicative effects in two-way analysis of variance. *Statist. Neerlandica 26*, 61–68.

Ghosh, M.N. and D. Sharma (1963). Power of Tukeys Test for Non-Additivity. *J. Roy. Statist. Soc. Ser. B 25*, 213–219.

Harter, H.L. and M.D. Lum (1962). An interpretation and extension of Tukey's one-degree of freedom for nonadditivity. *Aeronautical Research Laboratory Technical Report, ARL*, 62–313.

Johnson, D.E. and F.A. Graybill (1972). An analysis of a two-way model with interaction and no replication. *J. Amer. Statist. Assoc. 67*, 862–868.

Mandel, J. (1961). Non-additivity in two-way analysis of variance. *J. Amer. Statist. Assoc. 56*, 878–888.

Rasch, D. (1995). *Mathematische Statistik*. Berlin: Johann Ambrosius Barth, Heidelberg Leipzig und Wiley.

Tukey, J.W. (1949). One degree of freedom for nonadditivity. *Biometrics 5*, 232–242.

Tusell, F. (1990). Testing for interaction in two-way anova tables with no replication. *Comput. Statist. Data Anal. 10, 29–45.*