

# Rank correlation estimators and their limiting distributions

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## Abstract

We consider the following rank correlation problem: using independent copies of vectors  $(X, Y)$  and  $(X', Y')$ , where  $X, X' \in \mathbf{R}^d$  and  $Y, Y' \in \mathbf{R}$ , we want to predict, based on observations  $X$  and  $X'$ , whether  $Y < Y'$  or  $Y > Y'$  with maximal accuracy, i.e. we look for the ranking rule  $\phi$  that maximizes

$$E \mathbf{I}(Y < Y') \mathbf{I}(\phi(X) < \phi(X')). \quad (1)$$

We restrict to linear rules  $\phi(z) = \theta^T z, \theta \in \mathbf{R}^d$ . Han (1987) showed that a maximizer of the sample analogue of (1) is a consistent and asymptotically normal estimator of an unknown parameter. However, with respect to  $\theta$ , the function  $\mathbf{I}(y < y') \mathbf{I}(\theta^T x < \theta^T x')$  is not continuous, so calculating this estimator is computationally difficult. We propose (analogously to support vector machines in the classification theory; Vapnik, 1998) finding a minimizer of the following convex, with respect to  $\theta$ , function

$$\frac{1}{n(n-1)} \sum_{i \neq j} \mathbf{I}(Y_i > Y_j) \psi(\theta^T X_i - \theta^T X_j), \quad (2)$$

where  $\psi(z) = (1 - z)_+$ . There exist effective algorithms that allowed us to compute the minimizer of (2) (Bobrowski and Niemiro, 1984). We show strong consistency and asymptotic normality of the new estimator.

## Keywords

Rank correlation problem, Linear ranking rule, Discontinuous criterion function, Support vector machines, Convex minimization, U-statistics.

## References

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