

# Ordering multivariate distributions by entropy and variance

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## Abstract

This paper examines the role of variance and entropy in ordering multivariate distributions in terms of measure reduction. The entropy is a scalar measure of a distribution. On the contrary, the variance approach in dimension  $p$  leads to a matrix of covariance, and comparisons with entropy must repose on summary measures, such as the generalized variance, the determinant, or the total variance, the trace, of the matrix. Let  $\mathbf{X} = (X_1, \dots, X_p)$  and  $\mathbf{Y} = (X_1, \dots, X_{p-m})$  where  $p > m > 1$ , we compare the effect of dimension reduction from  $\mathbf{X}$  to  $\mathbf{Y}$  in terms of entropy and variance for several well-known multivariate distributions. We show that the concept of entropy reduction is more meaningful than variance reduction. Examples and typical cases are given to compare entropy and variance in assessing input variable importance in sensitivity analysis.

## Keywords

Entropy, Variance, Distribution ordering, Measure reduction, Sensitivity analysis.

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