On some matrix Diophantine equations

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Abstract

A lecture concerns the following matrix Diophantine equations

$$A^x + A^y + A^z = A^w, \tag{1}$$

$$A^x + A^y = A^z, \tag{2}$$

where $x, y, z, w \in N$.

In our paper *On some matrix Diophantine equations* we give necessary conditions for solvability of the equation (1) in some positive integers x, y, z, w under some restrictions for $A \in M_n(C), n \geq 2$ concerning eigenvalues of the matrix A.

Let $A \in M_n(C), n \geq 2$ is the matrix A which has at least one real eigenvalue $\alpha \in (0,1)$. If the matrix equation (1) is satisfied in positive integers x,y,z, w, then $\max\{x-w, y-w, z-w\} \geq 1$. If the matrix A has at least one real eigenvalue $\alpha > \sqrt{2}$ and the equation (1) is satisfied in positive integers x, y,z and w, then $\max\{x-w, y-w, z-w\} = -1$.

Moveover, we investigate the solvability of the matrix equations (1) and (2) for the non-negative real $n \times n$ matrices, where $|\det A| > 1$, in positive integers x, y, z, w for (1) and (2). Using the well-know theorems: Schur's theorem and Peron-Frobenius' theorem we obtain further results concerning solvability these equations.

Keywords

The matrix equations, Schur's theorem, Fermat's type Diophantine equation on matrices.

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