On some matrix Diophantine equations

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Abstract

A lecture concerns the following matrix Diophantine equations

\[ A^x + A^y + A^z = A^w, \]  
\[ A^x + A^y = A^w, \]  

where \( x, y, z, w \in \mathbb{N} \).

In our paper *On some matrix Diophantine equations* we give necessary conditions for solvability of the equation (1) in some positive integers \( x, y, z, w \) under some restrictions for \( A \in M_n(C), n \geq 2 \) concerning eigenvalues of the matrix \( A \).

Let \( A \in M_n(C), n \geq 2 \) is the matrix \( A \) which has at least one real eigenvalue \( \alpha \in (0,1) \). If the matrix equation (1) is satisfied in positive integers \( x,y,z,w \), then \( \max\{x-w, y-w, z-w\} \geq 1 \). If the matrix \( A \) has at least one real eigenvalue \( \alpha > \sqrt{2} \) and the equation (1) is satisfied in positive integers \( x, y, z \) and \( w \), then \( \max\{x-w, y-w, z-w\} = -1 \).

Moreover, we investigate the solvability of the matrix equations (1) and (2) for the non-negative real \( n \times n \) matrices, where \( |\det A| > 1 \), in positive integers \( x, y, z, w \) for (1) and (2). Using the well-know theorems: Schur’s theorem and Peron-Frobenius’ theorem we obtain further results concerning solvability these equations.

Keywords

The matrix equations, Schur’s theorem, Fermat’s type Diophantine equation on matrices.

References


