Regular A-optimal chemical balance weighing design

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Abstract

Regular A-optimal design there is design in which the sum of variances of estimators is minimal and attains the lower bound.
In the paper regular A-optimal chemical balance weighing designs are presented. There is considered the linear model $\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{e}$, which describe how to find unknown measurements of p objects using n weighing operations according to the design matrix $\mathbf{X}$ with elements $x_{ij} = -1,\ 0,\ 1$, $i = 1, 2, ..., n,\ j = 1, 2, ..., p$, $\mathbf{y}$ is $n \times 1$ random vector of the observed weights, $\mathbf{w}$ is $p \times 1$ vector representing unknown weights of objects. It is assumed that there are not systematic errors and they are equal correlated, i.e. for the $n \times 1$ random vector of errors $\mathbf{e}$ we have $\mathrm{E}(\mathbf{e}) = \mathbf{0}_n$ and $\mathrm{E}(\mathbf{e}\mathbf{e}^{'}) = \sigma^2\mathbf{G}$, where $\mathbf{0}_n$ is $n \times 1$ column vector of zeros, $\mathbf{G}$ is $n \times n$ positive definite diagonal matrix of known elements.
The problem is how, for a given matrix $\mathbf{G}$, determine the design matrix $\mathbf{X}$ in such a manner that the lower bound of sum of variances of estimators will be attained.
New construction methods, existence conditions and examples of the regular A-optimal chemical balance weighing design are given. The construction methods are based on set of the incidence matrices of the balanced bipartite weighing design.

Keywords

Balanced bipartite weighing design, Chemical balance weighing design, Optimal design.

References

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